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Interpretation of Anisotropy in the Cosmic Background Radiation [and Discussion]

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Interpretation of anisotropy in the cosmic background radiation

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We review mechanisms for producing temperature and polarization anisotropies in the microwave background radiation, and summarize their relation to the large-scale distribution of matter and to theories of galaxy formation. We also review possible sources of ambiguity in interpreting data, in particular the unknown opacity of the pregalactic gas and the possible contribution of discrete sources of radiation. Strategies for removing these ambiguities are discussed.

1. INTRODUCTION

The microwave background provides direct evidence of physical conditions at pregalactic epochs. Its high overall isotropy confirms that the geometry of the observable Universe can be closely approximated by a Robertson–Walker metric. But some small-amplitude fluctuations in background temperature would be caused by the irregularities that were the precursors of galaxies and clusters. Our aim in this paper is to discuss the temperature fluctuations on various angular scales that would be expected on the basis of various models for galaxy formation, and to assess the ways in which microwave background observations may help to discriminate between different models. We also examine the somewhat broader questions of what microwave background observations might tell us about the large-scale structure of the initial singularity, and about radiation emitted during the early evolution of the Universe, without making detailed assumptions about the formation of galaxies and clusters. Although we present a broad survey of the mechanisms for producing anisotropy, we do not discuss details of models that have received detailed attention in previous reviews. Our emphasis is primarily theoretical: current observations of anisotropy at angular scales of more than 1° are discussed by Fabbri *et al.* and Wilkinson in this volume, and observations at small angular scales are reviewed by Partridge (1982).

1.1. *The location of the last scatterers: the 'cosmic photosphere'*

The photons of the microwave background radiation (m.b.r.) have propagated freely since they were last scattered at a red shift z_* whose precise value depends on the post-recombination history of the matter, but which lies in the range $10 \lesssim z_* \lesssim 1000$. The present optical depth of a Hubble length due to Thomson scattering is $\tau_0 = x_e n_0 \sigma_T c H_0^{-1}$ where x_e is the degree of ionization, n is the total number density of electrons, σ_T is the Thomson scattering cross section, H is the Hubble parameter (equal to $100\,h$ km s^{-1} Mpc^{-1}) and a subscript 0 indicates the present value of a quantity. $\tau_0 \approx 0.06 \Omega_0 h x_e$, and is small even if $x_e \approx 1$. If $\Omega_0 = 1$ and $x_e =$ constant, then the optical depth to a red shift z is $\tau(z) \approx \frac{2}{3}\tau_0\{(1+z)^{\frac{3}{2}} - 1\}$. Let the nominal red shift of last scattering z_* be such that $\tau(z_*) = 1$. If $x_e \approx 1$ then $z_* \approx 10$, but if x_e is always lower than *ca.* $(\frac{1}{10}z)^{-\frac{3}{2}}$ (i.e. reheating is absent or too late) then we 'see' back to $z_{rec} \approx 1000$. If $z_* < 1000$ then the Universe gradually becomes transparent over an expansion timescale, and the effective thickness of the last scattering shell, determined by the width Δz_* of the function $(d/dz)\,e^{-\tau(z)}$, is *ca.* z_*. If, however, $z_* = z_{rec}$ then the Universe became transparent on a recombination timescale $\frac{1}{10}\times$ (expansion timescale), and the last scattering shell is relatively narrow, with $\Delta z_* \approx \frac{1}{15} z_*$. (Sunyaev & Zel'dovich 1970). These cases are illustrated in figure 1. In some models for galaxy formation where energy (and heavy elements) are generated at large red-shifts, other kinds of opacity may be competitive with Thomson scattering.

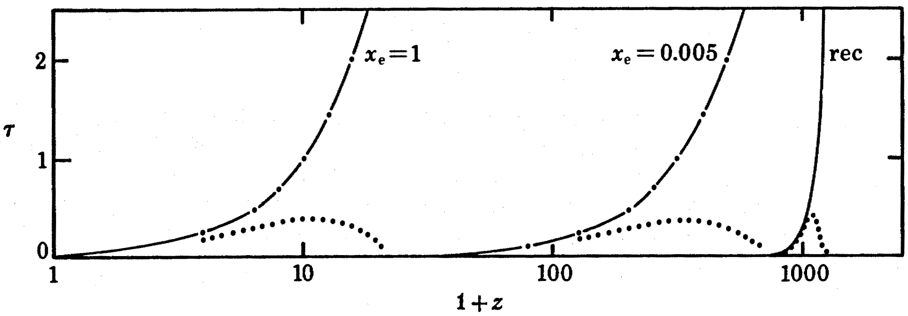

FIGURE 1. Optical depth $\tau(z)$ (dot–dash lines) and 'visibility factors' $\tau e^{-\tau}$ (dotted lines) for two values of fractional ionization x_e, assumed constant. Even a small amount of reheating is sufficient to produce a last scattering surface that is more 'smeared out' in red shift than the standard model of recombination (labelled 'rec').

1.2. *Angle–length relation*

Provided that $z_* > \Omega_0^{-1}$, which is so for almost all relevant models, the angle subtended by a given comoving region on the last scattering shell is essentially independent of z_*. A comoving length l_0 corresponds to an angle

$$\theta(l_0) \approx (\Omega_0/2c)\, l_0 H_0$$

and the mass contained in a sphere inscribed in a cone with angle θ is

$$M(\theta) \approx 4\times 10^{22}\Omega_0^{-2}h^{-1}\theta^3 M_\odot.$$

A region of present length 1 Mpc subtends an angle *ca.* $\frac{1}{2}\Omega_0 h$ arc minutes. A region that came within the horizon at z_* has present length $l_* \approx 2cH_0^{-1}(\Omega_0 z_*)^{-\frac{1}{2}}$ and subtends an angle $\theta_* \approx (\Omega_0/z_*)^{\frac{1}{2}}$.

Another important scale is the length corresponding to the intrinsic curvature of the spacelike hypersurfaces in the Robertson–Walker metric; this subtends an angle

$$\theta_c \approx \Omega_0(1-\Omega_0)^{-\frac{1}{2}}$$

if $\Omega_0 < 1$ (for $z_* \gg \Omega_0^{-1}$).

2. THE DIPOLE ANISOTROPY

The existence of a 24 h (dipole) anisotropy in the m.b.r. (Smoot *et al.* 1977) now seems well established. The likely main cause is our peculiar velocity within the local supercluster, with possibly some motion of the supercluster as a whole. The contribution to our peculiar velocity induced by small amplitude (linear) perturbations on large scales is

$$V_{pec} \approx (l/l_H)\,(\delta\rho/\rho)\,\Omega_0 c, \qquad (1)$$

where $l_H = cH_0^{-1}$.

We do not know enough about how $\langle(\delta\rho/\rho)^2\rangle$ falls off on large length-scales to be able to say what scale contributes most. As an illustrative example, a 5% density inhomogeneity on a scale of 100 h^{-1} Mpc would yield $V_{pec}/c \approx 10^{-3}\Omega_0$. One should therefore not be surprised if the value of V_{pec} implied by the dipole anisotropy disagrees with that inferred from the dynamics within the local supercluster (see White 1980).

The amplitude of the extrinsic dipole anisotropy is

$$\Delta I/I = (3-\alpha)\,(V_{pec}/c), \qquad (2)$$

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where α is the spectral index of the radiation. Thus the X-ray background ($\alpha \approx -\frac{1}{2}$) is roughly 3.5 times more sensitive than the Rayleigh–Jeans part of the m.b.r. for which $\alpha = 2$. One expects the X-ray dipole anisotropy to be additionally enhanced because the overdensity that provides our peculiar acceleration also presumably has an excess X-ray luminosity (Fabian & Warwick 1979). Because the peculiar velocity is smaller in a low density Universe a comparison of X-ray and m.b.r. anisotropy may constrain Ω_0. If the sources of the X-ray background did not evolve with z, then the X-ray dipole contribution due to emission from a nearby inhomogeneity of scale l would be *ca.* $(l/l_H)\,(\delta\rho/\rho)$, independent of Ω_0. An upper limit to the X-ray anisotropy can thus, in conjunction with an m.b.r. dipole measurement, set a lower limit to Ω_0. However, quantification of this limit depends on how the sources of the X-ray background evolve with z. (X-ray isotropy measurements may in fact provide more sensitive constraints than the microwave background on the lumpiness on scales 100–1000 Mpc (Rees 1981).)

The simple relation (2) must be modified if the perturbations are as large as the Hubble radius. On these scales, the best constraints on $\langle(\delta\rho/\rho)^2\rangle^{\frac{1}{2}}$ come from the limits on gravitationally induced fluctuations in the microwave background (see § 3). If, however, the Universe contains isolated 'lumps' on large scales, embedded in a much smoother general background, then, as Fabian & Warwick (1979) have pointed out, there may be detectable consequences for the X-ray background, for observations with currently attainable sensitivity, even though the influence of the 'lump' may be undetectable as far as the microwave background is concerned. The effect on the X-ray background can be analysed into three components.

(i) a 24 h effect due to the enhanced density of sources in the 'lump' (this effect would not exist if the lump lay beyond the red shift at which the X-ray background originates);

(ii) a 24 h effect due to our motion towards the 'lump' (this would be smaller than (2) because the sources of the background would themselves also be 'falling' towards the lump);

(iii) a 12 h quadrupole effect, with a minimum along the axis of symmetry towards the 'lump', due to the shear induced within the volume whence the bulk of the X-ray background originates.

Another useful consequence of (2) is that one may infer the spectral slope of the m.b.r. by measuring the dipole anisotropy at various frequencies: $(\Delta I/I)_\nu \propto (3-\alpha(\nu))$ (Danese & de Zotti 1980). This could provide a consistency check on claims of m.b.r. spectral distortions.

If the reported quadrupole anisotropy is indeed cosmological and due to the long-wavelength tail of a spectrum of inhomogeneity, then we expect an *intrinsic* dipole anisotropy of the same order as the quadrupole anisotropy, and then the velocity inferred from the m.b.r. would differ slightly (*ca.* 10%) from our actual peculiar velocity.

3. Intrinsic anisotropies of m.b.r. caused by gravitational effects

3.1. *Mechanisms producing anisotropy*

The most widely discussed mechanism for generating intrinsic anisotropy in the m.b.r. involves irregularities in the matter distribution on the 'cosmic photosphere' ($z \approx z_*$) where the radiation was last scattered.

3.1.1. *Large angle anisotropies ($\theta > \theta_*$, but excluding dipole)*

The dominant cause of anisotropy on these scales is the inhomogeneous gravitational potential on the last scattering shell (Sachs & Wolfe 1967). Because $\Delta z_* \lesssim z_*$ the last scattering

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shell is effectively sharp on these scales and so smearing is unimportant. A lump of amplitude $\delta\rho/\rho$ that straddles the last scattering shell produces

$$\Delta T/T \approx \Delta\phi \approx (l/l_*)^2 (\delta\rho/\rho)_{z=z_*}. \quad (3)$$

(Throughout this paper we define $(\Delta T/T)_\theta$ as the square root of the 'power' on angular scales between $\frac{1}{\sqrt{2}}\theta$ and $\sqrt{2\theta}$). Intervening lumps have a much smaller effect (Rees & Sciama 1968). For lumps larger than the R–W curvature scale (i.e. $\theta > \theta_c$), (3) ceases to apply. Gravity becomes dynamically unimportant and linear perturbations effectively freeze out at $z \approx \Omega_0^{-1}$, but the photons we see had then yet to emerge from the typical lump. For spherical lumps Kaiser (1982*a*) has found that $(\Delta T/T) \approx (\delta\rho/\rho)_0$ for $\theta > \theta_c$. For all $\theta > \theta_*$, $(\Delta T/T) \approx (\delta\rho/\rho)$ when the perturbation comes within the horizon (see also Peebles 1982). On scales exceeding the curvature length the simple power-law relations between length, area and volumes are not obeyed and so one may in principle use measurements of $(\Delta T/T)_\theta$ to distinguish between fluctuation hypotheses that have $(\delta\rho/\rho)$ going as some power of length, area or enclosed mass.

3.1.2. *Scales smaller than the particle horizon at* z_* (*i.e.* $\theta < \theta_*$)

At angles less than *ca.* θ_* then, if $\Delta z_* \approx z_*$, the fluctuations are due to $N \approx (\theta/\theta_*)^{-1}$ lumps of mass $M(\theta)$ along a line of sight. The fluctuations are then approximately (Sunyaev & Zel'dovich 1970; Sunyaev 1978)

$$(\Delta T/T)_\theta \approx N^{-\frac{1}{2}}(\delta\rho/\rho)_{z_*, M(\theta)}(l_\theta/l_*)^2. \quad (4)$$

The other possible 'general-relativistic' sources of anisotropy are non-perturbative departures from Friedmann models, such as homogeneous anisotropic cosmologies, or sourceless metric perturbations (e.g. gravitational waves). Also there is a possible contribution from perturbations much larger than the present horizon (see § 3.6).

3.2. *Temperature fluctuations from combined effects for* $z_* < z_{\text{dec}}$

Suppose the last scattering surface is 'smeared out', i.e. $\Delta z_* \approx z_* < 10^3$, and suppose that at this epoch there were density fluctuations $(\delta\rho/\rho)_{M, z_*}$ on scale M. Then the temperature fluctuations on an angular scale θ from the effects just described are

$$(\Delta T/T)_\theta \approx (\delta\rho/\rho)_{M(\theta), z_*} \times \begin{cases} (\theta_c/\theta_*)^2 & \theta > \theta_c \\ (\theta/\theta_*)^2 & \theta_* < \theta < \theta_c \\ (\theta/\theta_*)^{\frac{5}{2}} & \theta < \theta_*. \end{cases} \quad (5)$$

By inverting these formulae one obtains a limit on the present-day density fluctuation on scale l_0 from upper limits on $(\Delta T/T)_\theta$, once θ_* is assumed. At large angles $\theta > \theta_*$, this limit does not depend on z_* (because $\theta_* \approx (\Omega_0/z_*)^{\frac{1}{2}}$, and $(\delta\rho/\rho) \propto z_*^{-1}$ for $z_* > \Omega_0^{-1}$) but does assume that fluctuations were not *generated* at $z < z_*$:

$$(\delta\rho/\rho)_{l_0} = (\Delta T/T)_{\theta(l_0)}\Omega_0(1+z_*) \times \begin{cases} (\theta_*/\theta_c)^2 & \theta(l_0) > \theta_c \\ (\theta_*/\theta(l_0))^2 & \theta_* < \theta(l_0) < \theta_c \\ (\theta_*/\theta(l_0))^{\frac{5}{2}} & \theta(l_0) < \theta_*. \end{cases} \quad (6)$$

If $\Delta z \ll z_*$, fluctuations for $(\Delta z/z_*)\,\theta_* < \theta < \theta_*$ include several comparable effects (e.g. Doppler motions of electrons in the last scattering surface), which have been calculated for 'standard' recombination (Peebles & Yu 1970; Silk & Wilson 1980; Peebles 1981*b*, and references cited therein).

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3.3. *Theories for large-scale distribution of matter*

Various theories have been proposed for the form of $(\delta\rho/\rho)$, based on different assumptions about the initial conditions. These theories are best tested by observations of microwave anisotropy, especially at $\theta > \theta_*$, where scattering has little effect; until this can be done, their relative plausibility can be judged only on the subjective grounds of what assumptions seem most 'natural'. Alternative hypotheses include the following.

(i) The present-day distribution of mass density is random 'white noise' on large scales (Peebles 1981*a*).

(ii) The initial state of the Universe contained equal perturbations in gravitational potential (or space–time curvature) on all scales (Zel'dovich 1978).

(iii) The initial curvature fluctuations obeyed a featureless power-law over some mass range. Hypotheses (ii) and (iii) lead to 'adiabatic' or 'pancake' scenarios for galaxy formation.

(iv) There were no curvature fluctuations in the initial state, but only perturbations in entropy per baryon which obeyed a featureless power-law over some assumed mass range (Grishchuk & Zel'dovich 1978; Davis & Boynton 1980). This hypothesis leads to 'isothermal' or 'hierarchical' scenarios (Peebles 1980).

(v) A combination of (iii) and (iv) (Gott & Rees 1975; Silk & Wilson 1980).

(vi) The initial state of the universe was exactly uniform, and all perturbations resulted from instabilities of matter and causal processes (Layzer 1975, 1977; Ostriker & Cowie 1981; Ikeuchi 1981; Hogan 1982*b*).

The microwave background fluctuations for (i) to (v) have been calculated in the references given, with the assumption of 'standard' last scattering at z_{rec}.

For a given mass distribution, the reader can calculate the fluctuations expected for non-standard ionization history by using (5), although it is only natural to have $\Delta z_* \approx z_*$ in the isothermal models (iv) and (v), (see Sunyaev 1978; Hogan 1980), or the spontaneous one (vi). It is pertinent to summarize the main criticisms of each of the above.

To achieve a random white-noise distribution of matter *at present* (as in (i)) requires a contrived initial spectrum of perturbations in the hot Big Bang, because features are introduced into $(\delta\rho/\rho)$ on scales below $M_{\text{J}\gamma}$, the radiation Jeans mass *ca.* $10^{16-18}M_\odot$. However, this distribution does agree with observations of the present-day galaxy autocorrelation scale and the present observations of microwave anisotropy at *ca.* 6° and 90°. It is controversial whether the 'Zel'dovich' spectrum of adiabatic perturbations (option (ii)) can generate 'pancakes' at $z \approx 3$ to 5 without exceeding upper limits on microwave fluctuations at arc minute scales and at *ca.* 6°. However, the discrepancy is probably removed if the density of the Universe is dominated by massive neutrinos (Doroshkevich *et al.* 1980). 'Pure' isothermal fluctuations in entropy (option (iv)) only generate significant post-recombination density fluctuations up to $M_{\text{J}\gamma}$ (see § 3.5). Although this does not affect the formation of galaxies, etc., at $M \ll M_{\text{J}\gamma}$, it does imply that a featureless spectrum of pure isothermals could not produce a quadrupole or 6° anisotropy as large as claimed at present.

Hypotheses (iii) and (v), in some forms, can be criticized as unnatural, involving 'fine-tuning' at early times or the introduction of apparently arbitrary 'characteristic masses'. Hypothesis (vi) can be criticized mainly because no comprehensive model of this type has been demonstrated to work in the standard cosmology. It also has difficulties in producing large-scale perturbations in temperature, shown below.

3.4. *Temperature fluctuations for assumed theory of galaxy formation and assumed ionization history*

The expected small-scale fluctuations depend critically on the ionization history. In schemes where the initial fluctuations are purely adiabatic, the first bound systems to condense have

$$M = M_{\text{crit}} \approx 10^{14-16} M_{\odot} \quad (l \approx 10\text{–}30 \text{ Mpc}).$$

M_{crit} may be (*a*) the smallest scale of adiabatic perturbation to survive damping due to photon viscosity, or (*b*) the minimum scale on which neutrino fluctuations survive phase-mixing. (Here $M_{\text{crit}} \approx 10^{18}(m_{\nu}/1 \text{ eV})^2 M_{\odot} = (m_{\nu}/m_{\text{Planck}})^{-2} m_{\text{Planck}}$.)

In either scenario reheating occurs (if at all) too late for scattering to be important at $z < z_{\text{rec}}$. The implications of these schemes for the m.b.r. are reviewed by Silk & Wilson (1980), Peebles (1981*b*), Sunyaev (1978), Doroshkevich *et al.* (1978) and Doroshkevich *et al.* (1980). To form structure by the present, we need $(\delta\rho/\rho)_{\text{rec}} \approx 0.003$, and larger values if $\Omega_0 \ll 1$. We can then search for fluctuations on the few-arc-minute scale corresponding to M_{crit}. Adiabatic fluctuations are tightly constrained by observations with sensitivity 10^{-4} on arc minute scales.

Such, however, is not the case for isothermal models, where small systems begin collapsing $z \approx 1000$. If even a small fraction of matter collapses into stars or supermassive objects at this epoch, it may be sufficient to reheat gas to temperatures of a few thousand degrees and create a fractional ionization $x_e \gtrsim 0.005$ (say) sufficient to smear out the last scattering surface (see figure 1), and to reduce the small-angle fluctuations. The large-angle fluctuations depend mainly on the assumed initial spectrum (cf. § 3.3) and, if taken to be upper limits, do not seriously constrain the galaxy formation scheme. The 'minimum' fluctuations, even with an assumed white noise spectrum of initial entropy fluctuations, can be less than *ca.* 3×10^{-6} *at all angles* (see figure 3).

3.5. *Large-scale fluctuations from small-scale lumpiness*

One important question that arises is: what are the *large*-scale fluctuations generated by the presence of irregularities on small scales? A precise way to answer this question is to construct a model. Suppose we start with a Universe with no fluctuations on any scales, and pull matter into lumps on small scales such that momentum and energy can be transported only on scales smaller than about r. We then wish to calculate the large-scale fluctuations in the growing mode. A conserved quantity in such a mode is the Newtonian peculiar potential

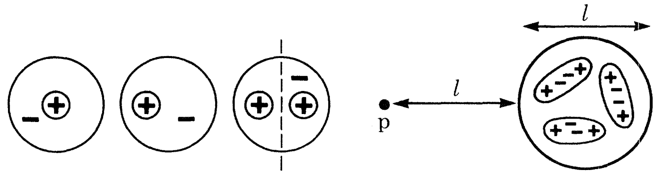
$$\delta\phi_l \approx (\delta\rho/\rho)_l \,(l/ct)^2$$

over a large-scale l, which gives directly the amplitude of the Sachs–Wolfe effect $(\Delta T/T)_{\theta} = \delta\phi_l$ on an angle $\theta = (l/ct)$.

Start with a single spherical region of radius r and mass m. If we make a spherically symmetric density perturbation (*a*) (in figure 2) it will have no effect on the gravitational field outside the region. On the other hand, we cannot make a perturbation of the form (*b*) because it would involve pushing against the walls of the spherical region; any perturbation involving forces confined to scales r must be plane-symmetric, of the form (*c*), because of momentum conservation. (In other words, if we managed to make a region like (*b*), it would necessarily be accompanied by a complementary neighbouring perturbation [+ −] [− +].) Although a random process would violate spherical or planar symmetry, the distribution of mass would, like (*c*), always be a *gravitational quadrupole*. That is, at large distances R from the region, the perturbation in ϕ is

$$\delta\phi(\mathbf{R}) \int_{\text{all space}} \mathrm{d}^3 r'\, \rho(\mathbf{r}')/(|\mathbf{R}-\mathbf{r}|) \approx (Gm/r)\,(R/r)^{-3} \quad (R \gg r). \tag{7}$$

Since in generating this quadrupole we have made no reference to the matter outside the spherical shell, nor indeed to any particular coordinate system, we are free to add a random distribution of randomly distributed quadrupoles, as in (d), which may even overlap each other. This is then a fairly general characterization of the large-scale gravitational effect of local



(a) (b) (c) (d)

FIGURE 2. Illustration of effect of small-scale lumpiness on large-scale temperature and density fluctuations. A superposition of gravitational quadrupoles (c) is produced by any random process that only transports energy and momentum over some scale r. (See text.)

forces acting on matter. The effects of the quadrupoles can then simply be added to find the value of $\delta\phi(\mathbf{R})$ at any point removed from all of them. If their orientations are statistically random, then a point P a distance l away from N quadrupoles has a potential

$$\begin{aligned}\delta\phi(l) &= N^{\frac{1}{2}}\,\delta\phi_i(R = l)\\ &\approx (Gm/r)\,(R/r)^{-\frac{3}{2}}\end{aligned}$$

or, in terms of mass,

$$\delta\phi_M \approx \delta\phi_m (M/m)^{-\frac{1}{2}},$$

which is an important result: the gravitational potential decreases on large scales. This corresponds to the famous '$M^{-\frac{7}{6}}$ law':

$$(\delta\rho/\rho)_M = (M/m)^{-\frac{7}{6}} \tag{8}$$

for growing modes of density perturbations (Zel'dovich 1965; Peebles 1974).

The mere presence of galaxy clusters, with $\delta\phi_m \approx 10^{-6}$ (i.e. $(\delta\rho/\rho) \approx 1$ on $l \approx 3h^{-1}$ Mpc, $(v/c) \approx 10^{-3}$), implies Sachs–Wolfe fluctuations that decrease with angular scale like $\theta^{-\frac{3}{2}}$. Any other behaviour (i.e. any anisotropy on $\theta > 1$ arc minute with $(\Delta T/T) > 10^{-6}$) requires non-local transfer of energy and momentum over large scales, or *acausal* initial perturbations, or a non-gravitational source of anisotropy.

(If we allow the quadrupoles to be aligned (which requires transmission of information but not energy) then

$$\delta\phi_R = N\,\delta\phi = Gm/r,$$

i.e. constant curvature fluctuations. This assumption is however not realistic for scales much larger than galaxy clusters.)

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A rigorous generalization of this argument to relativistic cosmologies requires the consideration of 'gauge' modes of perturbation, which are artefacts of one's coordinate system, a complicated subject treated by Bardeen (1980) and Press & Vishniac (1980). As an example, consider the case of 'pure' isothermal fluctuations, and no initial curvature fluctuations. Gradients in radiation pressure caused by entropy fluctuations can only transfer momentum on scales up to the Jeans mass $M_{J\gamma} = 10^{16-18}\ M_{\odot}$; the above argument would then imply that fluctuations on scales larger than $M_{J\gamma}$ should have the $M^{-\frac{7}{6}}$ spectrum. On the other hand, if we choose 'constant temperature' hypersurfaces, clearly the photons just red-shift away and leave us with density fluctuations with exactly the same spectrum as the initial entropy fluctuations: $M^{-\frac{1}{2}}$ for white noise. Which of these arguments is correct? It turns out that in the second case the putative density fluctuations in excess of $M^{-\frac{7}{6}}$ are 'gauge' fluctuations in the decaying mode, as shown by the following model.

Consider a $k = 0$ Universe, perfectly homogeneous before some t_1, at which time it becomes inhomogeneous owing to an energy-conserving, spherically symmetric process that creates a baryonic excess inside some region of radius R much larger than the radiation Jeans mass. After decoupling this is describable by a Bondi–Tolman solution (Bondi 1947). Conservation of energy requires that the *gravitational mass* M inside R is unchanged from the value it would have had if the perturbing process had not taken place. Thus, since the *proper mass* within R is enhanced, this matter must be on lower-energy (bound) orbits, the energy difference being $\delta\phi = (\Delta\rho/\rho)_B$, so the growing part of $(\delta\rho/\rho)_B$ is

$$(\delta\rho/\rho)_{B,\,\mathrm{growing}} \approx (ct_{\mathrm{eq}}/R)^2\,\delta\phi. \tag{9}$$

For perturbations that are still larger than the horizon at z_{eq}, this is much less than $(\delta\rho/\rho)_B$. If $(\delta\rho/\rho)_B$ has a Poisson form, then the growing part is $(\delta\rho/\rho)_B \propto r^{-\frac{7}{2}}$ as before.

3.6. *Perturbations due to inhomogeneities on scales outside the present horizon*

If $\Omega_0 \leqslant 1$ then our particle horizon will eventually grow to encompass an infinite amount of matter that we cannot yet observe. Inhomogeneities can induce gradients and shear across the region within our horizon, and may cause anisotropy in the m.b.r.

3.6.1. $\Omega_0 = 1$

Grishchuk & Zel'dovich (1978) found that a very long wavelength density ripple in a spatially flat Universe generates a quadrupole anisotropy

$$(\Delta T/T)_q \approx (l_{H_0})^2\,\nabla^2(\delta\phi) \approx (\Delta\rho/\rho)_0. \tag{10}$$

This may be understood in terms of photons becoming red-shifted or blue-shifted as they climb up or fall down the potential gradient associated with the perturbation. That there is no dipole term is because both we and the radiation are in free fall. If $\Omega_0 = 1$ the limits on any quadrupole constrain $(\delta\rho/\rho)_0$ to be less than about 10^{-4} on all scales outside the horizon (subject to the usual random phase assumption); however, the potential $\delta\phi$ may still diverge on scales larger than about $10^2 l_{H_0}$ and, since perturbations grow for all time if $\Omega_0 = 1$, these perturbations may have $(\delta\rho/\rho) \gtrsim 1$ when they come within the horizon.

3.6.2. $\Omega < 1$

The temperature perturbation due to a very large inhomogeneity in an open Universe has been estimated by Kaiser (1982*a*) by means of a spherical model. If the inhomogeneity has a radial scale ω (in units of the Robertson–Walker curvature scale) then $(\Delta T/T) \approx (1/\omega)\,(\delta\rho/\rho)$. Wilson (1982) and Fabbri *et al.* (1982) have calculated the temperature anisotropy for a very long wavlength plane wave in an open Universe and find that the fluctuations are dominated by harmonics with $l > 2$. This is analogous to the behaviour of homogeneous anisotropic cosmologies: if $\Omega_0 = 1$ the temperature varies as $\cos 2\theta$ (i.e. a quadrupole anisotropy), whereas if $\Omega_0 < 1$ then the anisotropy is concentrated in a region of angular extent *ca.* Ω_0 (Grishchuk *et al.* 1969; Doroshkevich *et al.* 1975).

4. Linear polarization

Linear polarization arises whenever radiation with a non-zero quadrupole moment of intensity is scattered. In addition to temperature fluctuations the microwave sky may therefore display linear polarization P_θ due to inhomogeneity and anisotropic expansion. The polarization of the photons that we receive tells us about the quadrupole anisotropy existing when the last scattering occurs.

Many authors (e.g. Rees 1968; Basko & Polnarev 1980) have shown that, in an anistropic but homogeneous cosmology, a degree of polarization comparable with the temperature anisotropy may be generated. The most favourable conditions prevail when reheating occurs at a moderate red shift $z_{rh} \approx 30$. A quadrupole temperature anisotropy builds up between z_{dec} and z_{rh}; polarization is generated and temperature anisotropy decays when this radiation is scattered off the re-ionized plasma.

When the early Universe is inhomogeneous, we would expect that $P_\theta \approx (\Delta T/T)_\theta$ on an angular scale corresponding to the thickness of the last scattering surface (i.e. $(\Delta z_*/z_*)\ \theta_*$). For perturbations on scales exceeding this, a typical last scatterer 'sees' only a small fraction of a wavelength, and the ratio $P_\theta : (\Delta T/T)_\theta$ falls as θ^{-2} for $\theta > \theta_*$. Measurements of this ratio could provide, in principle, a probe of the ionization history: in particular, if one found $P_{6°} \approx (\Delta T/T)_{6°}$ this would strongly condemn the adiabatic scenario in which the width of the last scattering shell subtends an angle of only *ca.* $3\Omega_0^{\frac{1}{2}}$ arc minutes.

Kaiser (1982*b*) has calculated the temperature and polarization patterns predicted in the adiabatic scenario. The r.m.s. polarization fluctuation is *ca.* 0.2 of the r.m.s. temperature fluctuations; both occur predominantly on scales less than about $3\Omega_0^{\frac{1}{2}}$ arc minutes. Temperature fluctuations on these angular scales are an inevitable consequence of the existence of clusters of galaxies, arising either from protoclusters at $z \approx z_*$ or from Compton 'cooling' of photons by hot gas in clusters (Sunyaev & Zel'dovich 1972); radio sources may also contribute $(\Delta T/T) \approx 10^{-5}$ to 10^{-6} (Longair & Sunyaev 1969). When temperature fluctuations are discovered, it may not be easy to determine their origin. But the polarization associated with the Sunyaev–Zel'dovich (1972) effect and from randomly superposed discrete sources would be well below the 'adiabatic' prediction. Polarization measurements may thus be able to discriminate among theories of galaxy formation.

5. Anisotropies of m.b.r. caused by sources of radiation

Probably the most significant cause of temperature fluctuations, apart from the gravitational effects discussed above, would be *discrete sources* of radiation. It is useful to consider three cases separately (for more details, see Hogan (1980, 1982*a*)).

5.1. *Discrete sources at* $z > z_*$

Depending on whether grains or other thermalizing agents were present, discrete sources of radiation would in general cause a spectral distortion of order Y, where Y is the fraction of the energy density that is non-primordial. If the radiation were released at $z > z_*$, it would be smeared out on small scales, but would not have travelled further than θ_* before being scattered into our line of sight. For $\theta > \theta_*$, and for sources whose properties are uncorrelated on large scales,

$$(\Delta T/T)_\theta \approx Y/N_\theta^{\frac{1}{2}} \quad (\theta > \theta_*),$$

where $N_\theta \propto \theta^2$ is the number of sources in a beam of size θ. For sources distributed at random with mean comoving separation l_s, radiating at *ca.* z_s,

$$(\Delta T/T)_\theta \approx \tfrac{1}{2}Y(l_s/ct_0)^{\frac{3}{2}}\,(1+z_s)^{\frac{1}{4}}\,\{\theta/(\theta_*^2+\theta^2)\}, \tag{10}$$

where $l_0 \equiv H_0^{-1}$. This is just the temperature fluctuation, peaked at *ca.* θ_*, obtained by filtering two-dimensional white noise at $\theta < \theta_*$. A frequency dependence could enter through $\theta_*(\lambda)$ if non-Thomson opacity were significant.

5.2. *Discrete sources at* $1 \ll z_s < z_*$

Sources of large red-shift this side of z_* would produce unfiltered two-dimensional white noise, and would necessarily produce a spectral distortion $Y(\lambda)$ that depends on the emission mechanism (e.g. molecular line emission, hot dust grains, free–free emission, or some non-thermal process). The anisotropy is

$$(\Delta T/T)_\theta \approx \tfrac{1}{2}Y(\lambda)\,(l_s/ct_0)^{\frac{3}{2}}\,(1+z_s)^{\frac{1}{4}}\,\theta^{-1}, \tag{11}$$

which would dominate over all the effects so far mentioned at sufficiently small θ (arc seconds) even for spectral distortions $Y(\lambda)$ that would otherwise be unobservably small. Also, if a distortion $Y(\lambda)$ is suspected to arise from radiation at high z, then (11) may be used to put constraints on the source distribution from anisotropy limits.

5.3. *Discrete sources at* $z_s \lesssim 1$

There is a qualitative difference between cosmologically remote sources at $z_s \gg 1$ and what might be termed 'contamination' from radio sources within the local Hubble volume. The $(\Delta T/T)_\theta$ from local sources comes from the brightest sources typically found in a solid angle θ^2 (Longair & Sunyaev 1969). The amplitude of $(\Delta T/T)_\theta$ is thus connected directly to the 'log N–log S' relation for radio sources:

$$(\Delta T/T)_\theta \propto \theta^{(2/\beta)-2}\,\nu^{-(2-\alpha)} \quad (\beta \leqslant 2), \tag{12}$$

where the source spectrum is proportional to ν^α and

$$N_\theta(>S(v)) \propto S(v)^{-\beta}$$

is the surface density of sources measured at v with flux more than $S(\nu)$. At the low flux levels relevant to this argument, the source counts would have flattened off to $\beta \lesssim 1$, yielding $(\Delta T/T)_\theta$ almost independent of θ.

The amplitude of fluctuations from 'local' sources depends strongly on wavelength. At $\lambda = 11$ cm, fluctuations have already been observed that may be attributed to sources. (Martin *et al.* 1980). At shorter wavelengths the luminosity function of radio and infrared sources is too uncertain to permit an accurate estimate of $(\Delta T/T)_{\theta,\lambda}$, but it is unlikely to be less than about 10^{-6} for any θ, λ (see Danese *et al.* 1982).

6. Conclusions

Observations are tantalizingly close to the sentitivity level where they may reveal a wealth of data on m.b.r. anisotropies: this seems true now, as it has done for the last decade. Such data could yield crucial evidence on the processes whereby bound systems condensed out after the era of recombination, and on the nature of the initial fluctuations. The possible 'scenarios' vary widely: the formation of sub-galactic masses (and an associated energy input) could start at $z \approx 1000$; on the other hand, in the purely adiabatic model, everything remains quiescent until cluster (or supercluster) masses condense out at $z \lesssim 5$. In this paper we have tried to emphasize that the interpretation of any $(\Delta T/T)_\theta$ measured may be ambiguous. Theorists have generally focused attention on the case where we are seeing perturbations induced by

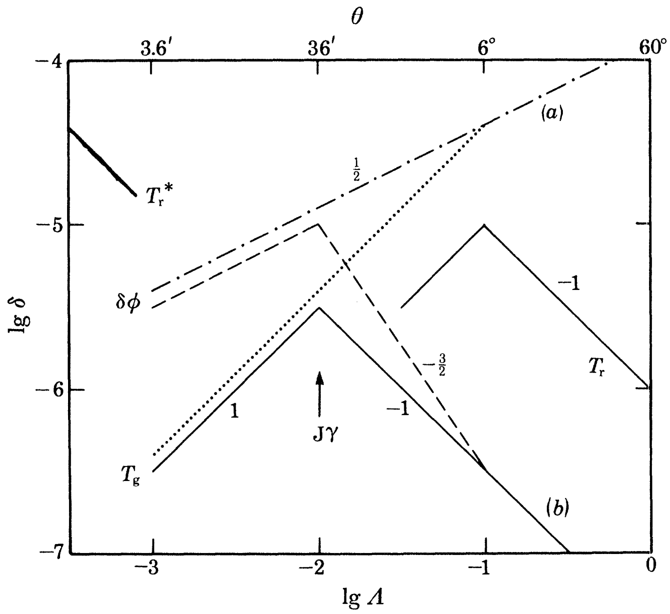

Figure 3. Fluctuations in gravitational potential $\delta\phi$, and in temperature T, as a function of angular scale θ or corresponding comoving wavelength $\Lambda \equiv l/ct_0$. For each case here, $\Omega = 1$ and $\theta_* = 6°$. Potential fluctuations $\delta\phi$ are shown for two distributions of matter: (a) white-noise fluctuations $(\delta\rho/\rho) \propto M^{-\frac{1}{2}}$ extending to arbitrarily large scale (Peebles 1981), and (b) white-noise fluctuations $(\delta\rho/\rho) \propto M^{-\frac{1}{2}}$ with a cut-off above the radiation Jeans mass $M_{J\gamma}$, as produced by 'pure' primordial entropy fluctuations (Grishchuk & Zel'dovich 1978) or by energetic processes at $z \gtrsim 100$ (Hogan 1982*b*). Approximate temperature fluctuations due to classical gravitational effects in both models (T_g) are plotted with last scattering at $z_* \approx 100$, $\theta_* = 6°$. Curve labelled T_r shows the approximate angular spectrum of anisotropy produced by discrete sources of radiation at $z > z_*$; curve labelled T_r^* shows the angular spectrum of anisotropy produced by discrete sources of radiation at $1 \ll z < z_*$. Lines are labelled by logarithmic slopes.

potential fluctuations or peculiar velocities, or both, at the recombination era and uncontaminated by the effects of early reheating, i.e. $z_* \approx 1000$, $\Delta z_*/z_* \approx \frac{1}{15}$. But in alternative models where $z_* < 1000$, fluctuations on arc minute scales would be smeared out; conceivably, the fluctuations on all scales could (consistently with galaxy formation) be only of order 10^{-6}.

It remains important to pursue studies on both large and small angular scales. We emphasize specifically, in conclusion, the following points:

(i) *Large angular scales* ($6° \lesssim \theta \lesssim 90°$). These fluctuations, exceeding the horizon size at z_*, would be most naturally attributed to inhomogeneities imprinted 'acausally' or generated by exotic processes in the very early Universe. Confirmation of real effects at the *ca.* 10^{-4} level would thus be of fundamental importance. The initial spectrum could be inferred from the θ dependence of $(\Delta T/T)$, almost independently of the thermal history of matter at $z \lesssim 1000$; even for an intrinsic 'power-law' spectrum, the behaviour of $(\Delta T/T)$ on scales larger than θ_c (the angle corresponding to the Robertson–Walker curvature radius) would yield clues to the underlying mechanism. Although some level of large-scale anisotropy could be generated by secondary processes (see § 5) these would always tend to decrease with θ: if $(\Delta T/T)_\theta$ were found to increase with θ for $\theta_* < \theta < \theta_c$, this would imply that our Universe contained 'acausal' curvature fluctuations.

(ii) *Dependence on observing frequency.* Anisotropies on arc minute angular scales relate directly to the masses of clusters of galaxies (where any gravitational instability model requires $(\delta\rho/\rho) \approx 0.003\Omega_0^{-1}$ at $z = 1000$). But these anisotropies would be smeared out if early reheating occurred; moreover, any anisotropies of 'secondary' origin would confuse the interpretation. If small-scale effects were detected at one wavelength, it would be valuable to scan the same strip of sky at another wavelength. If we are seeing back to $z = 1000$, the temperature fluctuations at different wavelengths should 'track' each other precisely. Any secondary emission (due to discrete sources or dust) would yield λ-dependent effects. Also, if the last scattering surface were at $z_* < 1000$, it might involve other λ-dependent opacities, additional to Thomson scattering; observations at different wavelengths would then be sampling different depths, so no detailed correlations would be expected in any $(\Delta T/T)$ measured on small angular scales.

(iii) *Contamination from within our Galaxy.* Wilkinson (this symposium) emphasizes that the precision of quadrupole anisotropy measurements is bedevilled by the contribution from our Galaxy: this is comparable with the intrinsic quadrupole that has been claimed; furthermore, even if there were no intrinsic quadrupole, this contamination would restrict the sensitivity of the observational uppper limits that could be set. Moreover, even at high galactic latitudes, and on small angular scales, patchy emission from interstellar HII or dust may be troublesome at the *ca.* 10^{-5} level. Consequently, it would be useful to scan separate small areas of sky with widely different galactic coordinates. Any discrepancies between the statistics of $(\Delta T/T)_\theta$ (for small θ) in these areas would be attributable to galactic contamination.

(iv) *Polarization.* Searches for linear polarization, if carried out with similar sensitivity to the $(\Delta T/T)$ measurements, can provide independent evidence for anisotropies. Moreover, the ratio of $(\Delta T/T)_\theta$ to P_θ is sensitive to the nature of the last scattering surface: in particular, if these quantities were comparable on angular scales larger than about 2°, the 'standard' model with $z_* \approx 1000$, $\Delta z_*/z_* \approx \frac{1}{15}$ could be ruled out (see § 4).

(v) *Isotropy measurements in other wavebands.* The X-ray and γ-ray bands are the only ones apart from the microwave band where the extragalactic background is not swamped by emission or absorption in our Galaxy. The X-ray background isotropy on scales 3–30° is a sensitive probe of lumpiness in the matter distribution on scales 100–1000 Mpc; moreover, the origin of the dipole microwave anisotropy can be clarified by comparison with the amplitude of (or limits to) a corresponding anisotropy in X-rays (see § 2).

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Discussion

R. Fabbri (*Istituto di Fisica Superiore, Università di Firenze, Italy*). Might the large-scale anisotropy of the cosmic background be *strongly* frequency-dependent and nevertheless be of extragalactic origin?

M. J. Rees. If reheating occurred in a 'lumpy' fashion when galaxies formed, then one might indeed expect angular fluctuations, which would be strongly frequency-dependent if the anisotropic emission were due to bremsstrahlung or to dust. (Note also that there would not necessarily be a detailed correlation, even in *sign*, between the fluctuations at different frequencies from the same strip of sky, because the 'cosmic photosphere' would not be at the same red shift at all frequencies if the emission and opacity were frequency-dependent.) There could even be a quadrupole fluctuation generated in this way; however, if the fluctuations are produced in a causal way then, as Hogan has discussed, the amplitude should fall off roughly as θ^{-1} rather than increasing with θ.

W. H. McCrea, F.R.S. (*Astronomy Centre, University of Sussex, U.K.*). Might one not regard the problem of 'acausal' homogeneity in the early Universe as maybe even more difficult than that of acausal fluctuations?

M. J. Rees. I completely agree. Dr Guth's paper discusses one possible explanation for the overall homogeneity. If this turns out not to work, then (given that the 'mixmaster' approach seems also to fail), the resolution of this fundamental mystery must probably be sought by investigating the quantum gravity era ($t \approx t_{\mathrm{Planck}} \approx 10^{-44}$ s), when the (classical) concept of the particle horizon would be transcended.

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